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Mathematics News Letter

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A Challenge to Forward-looking Mathematics Teachers in the Colleges and High Schools of Louisiana and Mississippi.

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RESPONSIBILITY IS OPPORTUNITY

When a man of the ripeness and wisdom of George B. Olds, now President Emeritus of Amherst College, lifts his voice to tell the world that the study of mathematics promotes English style by pruning away from it diffuseness and obscurity, it is time that college administrations hesitate before removing all mathematics from the list of requirements for a liberal arts degree.

It is to be doubted if the healthy minded youth can be found who is incapable of being lead by proper teaching to some degree of success in mathematical study. In proposing this we assume that healthy-mindedness implies at least a measure of native ability to reason. If these assumptions are correct, in the light of the undisputed fact that exercise in mathematics implies exercise in reasoning, it is difficult to escape the conclusion that school administrations should pause before accomodating their courses to the idea that some students are mathematical-minded and some are not. Our irritation increases from year to year as we listen to such complaints as, "I have no head for mathematics," "I can eat up languages but I am a dumbbell when it comes to math." "My history teacher says he could never do a simple example in arithmetic." "Why should I

take college algebra which I despise and for which I shall never have any use?"

Remedies? To diagnose is generally easier than to cure. Scores of books have been written on the teaching of mathematics. The crying need of the time is not more books on teaching, but, rather, more inspiration and more motivation of young minds to a living interest in mathematics. Such motivation must in the main come from two classes of workers, namely, the high school mathematics teacher and the teacher of freshman mathematics. This is evident when we reflect that for the average youth the high school and freshman years are largely deciding years in determining permanent mental attitudes. Thus is responsibility laid heavily on high school and college freshman instruction. A half-prepared secondary teacher struggling **ineffectively to put** over a geometry lesson to a class of twenty motivates and inspires no one. To the freshman with ability in mathematics—but needing to have that ability discovered and nurtured—there appears no rainbow of mathematical promise in a scheme which places his mathematical guidance in the hands of a raw graduate student for exactly one-fourth of his college life, while the real masters and matured teachers—those capable of revealing to a hesitant young mind the glory and the charm of mathematics afford him no contacts.

Great is our responsibility! Great is our opportunity!

—*S. T. S.*

MATHEMATICS IN EVERY THING

The enormous increase in industrial and scientific specialization which has taken place in the last quarter of a century has resulted in greatly increased demands on mathematics. This fact has strongly impressed the intelligent lay mind. (See in this issue of the News Letter a quotation from an editorial of the Saturday Evening Post.) The evidences of it are found on every hand. For instance, the opening paragraph of the preface of a recent text for chemistry students* is as follows: "Science is best expressed when it is mathematically expressed; and yet most students of natural science cringe at the pros-

pect of higher mathematics. It is because the author has seen so many students struggling against the handicap of an inadequate preparation in mathematics—spending more effort in avoiding mathematics than needs to be spent in learning it—that this book has been written." In similar manner one field after another is being compelled to turn to mathematics for that precise formulation of its basic principles which is essential to its ranking as a science or near-science.

In singular contrast with this rapidly growing demand for the technique of mathematics in scientific and quasi-scientific fields is the apparent attitude of indifference to it on the part, not only of many of our college-of-liberal-arts groups, but also of large numbers of those whose very choice of a major subject predestines them to the use of much mathematics in order to proper success in their major. Happily for the cause of mathematics, the latter group early discovers that any compromise which permits a basic tool to be unused makes that success questionable. On the other hand the case for mathematics as a valuable part of a proposed language or social science course has yet to be made out before the bar of opinion in a considerable number of our American institutions.

We seriously doubt if one per cent of the trained mathematicians of America question the great value of mathematical study as a mental discipline. The fact that probably less than one-fourth of one per cent of the membership of the Mathematical Association of America and the American Mathematical Society would be found questioning such a value, could at best be made to influence the construction of school curricula in considerably less degree than curriculum building has already been influenced by ideas, centering about "formal discipline," which have been fostered in the last quarter of a century by certain schools of education. The reason which is ordinarily advanced to account for this is that a mathematician is naturally if unconsciously biased in favor of his own field. On the other hand it should be evident to all fair-minded investigators that the existence of such a bias cannot justify a wholesale throwing overboard of the weight of EXPERIENCE.

—*S. T. S.*

*"Mathematical Preparation for Physical Chemistry," by Farrington Daniels, Associate Professor of Chemistry, University of Wisconsin.

TO THE HIGH SCHOOL MATHEMATICS TEACHERS

Every high school teacher should be vitally interested in promoting the cause of mathematics. Professor H. E. Slaught, University of Chicago, in his article, "Mathematics and Sunshine," May issue of the Mathematics Teacher, has this to say: "It is our business as teachers to sell mathematics to the public. To do this we need to believe in it most thoroughly ourselves, to proclaim it on every reasonable occasion, such as assembly talks, parent-teacher discussions, club meetings, etc., and to inspire our pupils with propaganda that mathematics is the most marvelous and most powerful achievement of the human race." He further states that in order to accomplish this we should combine our efforts in such an organization as the National Council.

The Mathematics News Letter, official organ of the Louisiana-Mississippi Branch of the Mathematical Association of America and National Council of Teachers of Mathematics, is seeking such combination of efforts.

If we can get every high school teacher of mathematics in Louisiana and Mississippi to become readers of and subscribers to and contributors of material to the Mathematics News Letter there will follow a wonderful growth of mathematical interest and friendship among the teachers of this section. I am requesting that each high school teacher who is a reader of the News Letter assist in bringing to the attention of other teachers the work and value of the News Letter.

All high school material should be sent to Henry Schroeder, Ruston, Louisiana.

—*H. S.*

DEGREES AND DIPLOMAS

(From a Saturday Evening Post Editorial)

Many a bright and promising college man drops his studies along with his athletics. After a few years he takes on weight and becomes heavy on his feet. His intimates make teasing remarks about bay windows; but none will have the hardihood to hint that he has likewise developed a bay window of the mind or has allowed his mental machinery to rust and jam through sheer neglect and shiftlessness. Faithful are the

wounds of a friend, but friendship is the price of inflicting them.

Hopeless cases of fine minds gone soft and flabby are so common that it is not too much to say that arrested intellectual development is the great national disease of our educationally privileged classes. Sheer lack of will power and mental stamina makes it difficult or impossible for us to forgo ease and rest and attack irksome tasks such as reading the books that harden the brain, but which are so new and strange that they must be studied as a child studies geometry—painfully and doggedly. Since men now in their fifties went to college the whole universe has been taken down and reassembled in a new and unfamiliar form. Literature, relatively speaking, has been marking time. Science has been going ahead by running leaps. Unfortunately for the casual and easily daunted reader, modern science is written in the language of mathematics and in the dialect of calculus; not only physics, chemistry and electricity but physiology and the other life sciences. Lack of easy familiarity with higher mathematics is a formidable obstacle between our ignorance and any real grasp of the modern conceptions of the universe we live in; and that obstacle will continue to bar our paths until the extraordinary importance of mathematical studies receives full and practical recognition.

CREATING AN INTEREST IN GEOMETRY*

By MRS. B. A. SUMMER
Lumberton, Mississippi

The students in the secondary schools of today are demanding to know why they study geometry. Before any formal study of the subject is made many think it is required in the curriculum for the sake of mental discipline solely. It is our duty to show some of the practical values of geometry to the student who is just beginning a study of the subject. We may create an interest in this subject by showing what a very important part geometric principles play in everyday life.

In contact with the material world the students unknowingly gain a great fund of geometric facts. It is necessary to call their attention to these facts and lead them to observe similar facts. We should show them how geometry is closely related to the other sciences and how it has played an important

part in the advancement of civilization and at the same time prove to them that the knowledge of geometry brings pleasure and increased capacity for enjoyment by developing an appreciation for the beauty and utility of architecture, art, and engineering constructions. Students learn by experimentation, therefore let them observe the laws of geometry operating in concrete form before they are required to do logical thinking.

Construction work leads to investigation. Even though the constructions may require many hours of tedious work, outside work of such a fascinating kind seldom fails to awaken the interest of the students. Constructions that are simple, well organized and purposeful develop skill in performance and satisfaction in accomplishment. It is a natural instinct in children to see how things are made and put together. Of course the constructions at first must necessarily be very simple. Later require them to bring a construction of architectural design or a complicated linoleum pattern in use in your city. Many students will copy church windows. Students will take great pride in their notebooks and will construct many original figures.

After introducing the simplest facts of geometry to a class it is necessary to teach the students how to study, since the language is quite different from that with which they are familiar. Correct methods of studying help to arouse the interest of the student. Unless students are interested in a subject they do not put forth their best efforts. One of the purposes of the teaching of geometry is to develop self-reliance and the power of initiative. Since most students like to argue, we may use this fact to an advantage by requiring them to base their arguments of geometric principles only on sound reasoning. We are then teaching them the force and value of concise statements.

In our own experience an illustrated theme on "The Uses of Geometry in Ancient and Modern Times" proved a help in creating an interest. The themes varied in content but on the whole were excellent. Some of the students arranged the pictures which they used as illustrations so as to form some geometric figure. Throughout last semester reports were given by students on the work contributed to geometry by famous mathematicians. This semester they have outlined an ancient and medieval history of geometry and will later outline the

modern history. In connection with the history the students are required to make marginal notes in their textbooks next to the theorems telling certain interesting facts pertaining to them.

Occasionally have a review of a book conducting it in the manner of a contest. On the day before the contest two leaders should be selected who in turn choose the other members of the class. The students should write the questions on uniform slips of paper and hand these in so that the teacher may see that there are no duplicates. On the day of the contest place these on the desk face down and allow the students to draw the slips. Put the figure of the problem, or theorem, on the board and write out the "given" and the "to prove". Two points may be given for the figure, one point for the construction, one point for the "given," one point for the "to prove" and five point for the proof. In order that the contest may be continuous each group should keep a student at the board. At the end of the recitation the scores should be added and the side scoring the highest total number of points should be declared the winner. The students feeling the keen competition are aroused to do their best.

If it were possible for you to present each year one or more plays relating to mathematics, similar to the one which has just been so excellently presented you will find that the students will become interested in their work. Require the students who take a part in the plays to make a certain grade, for instance an average of at least 85%. The plays should be presented before the general assembly of the student body.

There are of course many ways in which to interest a class in the subject of geometry. I have dealt only with those which I have tried out in my own classes.

*Read before the Jackson (Mississippi) joint meeting of the La.-Miss. Section of M. A. of A. and La.-Miss. Branch of the National Council of Teachers of Mathematics.

THE NATIONAL COUNCIL YEARBOOKS

The first Yearbook of the National Council may be secured for $1.10 from C. M. Austin, Oak Park High School, Oak Park, Ill. The second one may be secured for $1.25, and the third one for $1.75.

NEEDED READJUSTMENT BETWEEN THE TEACHING OF MATHEMATICS IN THE HIGH SCHOOLS AND THE COLLEGES

By W. C. ROATEN
DeRidder, La.

Within the past fifty years many important changes have taken place in educational theory and practice in this country, and today we find ourselves trying to make many adjustments to suit these changes. Formerly only the strongest mentally attended high school, but now we find that it is the aim of practically all pupils to graduate from high school, and of those who do graduate a larger and larger per cent desire to go to college. Thorndyke says that forty years ago one pupil in ten graduated from high school, while now this percentage has increased to one in three. In the states of Louisiana and Mississippi one-half of the high school graduates enter college.

But with this increase in the number demanding admission to college has come a decrease in the average mental ability of the applicants; and, since the standard of college entrance requirements was set to suit the former mental average, it follows that the number of failures of college freshmen is much larger than it should be. The problem, then, seems to concern an adjustment to meet this condition. We must recognize that the young men and women who have the ambition to go to college, but whose I. Q's are below the required standard, are entitled to consideration, and that it is the duty of the high school and college authorities combined to meet this requirement. There is no good reason why fifty per cent of the young men and women of Mississippi and Louisiana who enter college should fail.

Since mathematics, next to science, seems to be the cause of these failures, it must be the particular province of the mathematics teachers to make some contribution to the solution of the problem.

Of course it is recognized at the outset that secondary teachers are at a great disadvantage because they are compelled to accept whatever entrance requirements the colleges may fix for the high school graduates. While in theory it is true that as a rule these requirements are not too heavy, it is a fact that the

entrance requirements are not always an index of what the student must be able to do in order to remain in a given class. The college professor, subject to no regulations in the premises, often makes the gap between what the pupil learns in the secondary school and what he requires in college so great that it is quite impossible for him to make the transition. Too much depends on the methods and personality of the college professor. It is possible for the live high school mathematics teacher to meet the college requirements unless those whose I. Q's are too low are included; but it must be understood that such subjects as the Binomial Theorem, Progressions, Logarithms, and Trigonometry are not to be included.

But the necessary conditions are not being met on either side. Some reasons for this may be given: Probably the most important is the lack of competent and wide-awake teachers. Too little care is exercised by the school authorities in selecting members of the high school faculties. Then, the college demands are not always understood by the secondary teachers. These teachers may be both competent and willing, but on account of the varying and indefinite requirements of the colleges, pupils are permitted to leave the high school for college with very hazy ideas as to what will be required of them. Another cause of failure is the lack of cooperation between the colleges and the secondary schools. Little effort has heretofore been made to come to a mutual understanding of the common difficulties to be met and overcome. A sympathetic attitude on the part of college professors would help many a poor bewildered freshman to get his feet firmly on the ground. Going from his home school where his whims and weaknesses are known he easily loses his head when thrown into a hopper with dozens of others in like condition. Another source of trouble is the lack of coordination between college and high school in the matter of fixing the high school course of study. These courses and the curricula of the college seem to be arranged with only the upper one-fourth of the pupils in mind, whereas the time is now here when the second and third fourth are demanding the privilege of attending college.

In the Mississippi course of study only one unit of mathematics is required for graduation from the high school, under which arrangement no one can enter college without looking out for enough electives in mathematics to allow him to meet the

entrance requirements. The Louisiana course requires three and one-half units of mathematics before a high school diploma can be awarded. One of these apparently assumes that no one from the state high schools will enter college while the other assumes that all high school graduates will want to enter college, Of course both assumptions are untrue. There are certain steps which each should take in order to meet the conditions more satisfactorily.

The time has come when educational institutions must recognize the different mental abilities of high school graduates. We can no longer recognize the upper fourth of the class and relegate the lower three-fourths to oblivion. In thus taking these differences into account, requirements for high school graduation must be graduated to fit the individual abilities. On the college side, the entrance requirements must be so elastic as to permit the entrance into college courses of every boy and girl who graduates from high school.

Education is becoming—and it must be—democratic in practice as well as in theory. Democracy in education means "equal chances for unequal minds." This means, more educational institutions with a much wider range of curricula. To show that this thought is prevalent among educators, the following quotation from the resolutions committee of the National Educational Association at a recent meeting is given: "Higher educational institutions will have to continue to make broader adaptations to individual differences among increasing numbers."

In view of the foregoing, the following recommendations are made: 1. High school and college administrators are requested to seek a middle ground on which both may stand in an effort to smooth the way for the college entrant. 2. The colleges should accept and act favorably upon the recent recommendation of the Committee on College Entrance Requirements in which it says: "That the uniform entrance blanks should be so revised as to contain full and complete information of a student's secondary school record, including record of failures, with provision whereby the principal may certify if the applicant is in the upper-half, upper-third, or upper-fourth of the graduating class."

The Louisiana Academy of Science holds its next meeting at Lafayette conjointly with that of La.-Miss. Section of M. A. of A.

SOME RESULTS OF INDIFFERENT TEACHING OF HIGH SCHOOL MATHEMATICS

J. W. HAMMETT
Former High School Principal

Much has been, and is being, said about methods of teaching. Books are written, classes are conducted, and even schools maintained in order that better methods may be used in our elementary and secondary schools. The purpose of this article is not to suggest any specific method, but to try to show where some of the results of poor teaching of mathematics are very outstanding in the subsequent work of the pupil.

Very naturally, it might be thought that the most outstanding results would manifest themselves in the freshman year of college mathematics, and, most assuredly, they show up there. This, however, is not by any means the only place. How many times has the teacher of high school physics, when trying to teach the law of "falling bodies", found that the pupil who has a unit in high school algebra is entirely lost when he is asked to solve for t in the formula, $S=\frac{1}{2}gt^2$. Again, ask the pupil who has had both algebra and geometry to find the resultant of two forces acting at an angle, where it is desired to find the hypotenuse of a right triangle with the sides given, or, when the angle is not a right angle, to complete the parallelogram and find the resultant. The number who will not even know how to start the solution is alarming. The writer recalls a class of thirty-two in physics, where only two could apply the first method and none the second. This weakness is not less evident in exercises in chemistry involving simple ratio and proportion, to say nothing of the more complicated exercises in chemistry and physics. I have noticed the same results in general science, zoology and biology. In all of these subjects I have repeatedly met with many who were unable to apply the simplest formulas of arithmetic, algebra or geometry.

How can such conditions be remedied? It is a question that should concern every mathematics teacher. One thing is evident: the remedy can never be found by the high school teacher trying to shift the responsibility to the elementary teacher, then haphazardly pushing the student through high school, only to have the college professor shift it to the high

school teacher. If criticism is justified, and in too many cases it is, such a "shift" policy does the students no good, only creates in them a disgust for college life, and fails. There is one way to attack the problem, and that is for all teachers of the subject, from the grammar school to the college, to treat it as a mutual one. When a teacher is confronted with a specific difficulty, let that teacher, through a common medium, ask for helpful suggestions. Likewise, if there are those who have overcome some difficult situation, let them give others the benefit of their experience. I once heard a classmate, a very energetic young teacher, ask our training teacher, "How can I best secure the best cooperation of my patrons?" His reply was, "Know your patrons and cooperate with them." Could not this be applied to our own case? Let us know each other and cooperate.

SUMMER CLASSES IN MATHEMATICS AT LOUISIANA COLLEGE, PINEVILLE, LOUISIANA

By C. D. SMITH
Head of Department

As a whole the summer work in the department has been very gratifying. The courses included classes in the Calculus, Trigonometry, and Algebra. Thirty hours per week were scheduled for each term and the classes averaged from ten to sixteen. The number is unusually large for summer classes in mathematics and the character of work accomplished may be judged by the fact that only four failures occurred among freshmen during the first term. We attribute the high percentage of satisfactory work to carefully planned review lessons with which we introduce our first courses. Almost without exception our classes were composed of regular students and teachers who are working toward degrees. It is a hopeful sign when we have numbers of teachers come to us and say, "I will have classes in algebra or geometry next year and I want to study mathematics with a view to knowing more about the subject." I think we should be busy asking such teachers to line up with The Association or The Council.

Courses for the Fall Term will include Algebra, Geometry, Trigonometry, Analytics, and Calculus. We will use the Iowa Placement Test in mathematics together with an Aptitudes Test

as a basis for placement. The method of placing high, average and low is used here and we offer rewards to those who prove worthy by promoting them to higher levels. Those who finish low are not permitted to schedule elective courses. We expect larger classes this year in the elective courses as we will be carrying at least twelve majors and a number of minors besides the usual number who elect courses for various reasons. There seems to be a tendency here on the part of the more intellectual type to turn toward more mathematics in their schedules. We are pulling hard for it in the firm conviction that their education will be enriched and their mental powers enhanced by just that sort of schedule.

CONCERNING GRAPHICAL ALGEBRA

By A. C. MADDOX
State Normal College

The question as to the relative amount of time that should be devoted to graphical methods in high school algebra is a question concerning which there seems to be a very wide range of opinion among teachers of the subject. Much of the confusion is no doubt due to the lack of agreement as to the purposes for which students should do graphical work in algebra. This lack of agreement in purpose naturally leads to wide differences in opinion regarding such relatively minor questions as those pertaining to the number of isolated points of a locus the student should plot before constructing the locus, and to the mechanical precision with which he should be expected to construct the locus, or graph, of an algebraic equation.

Admitting that there are other worthwhile purposes to be accomplished by the use of graphical methods in algebra, the writer of this article holds strongly to the opinion that the central purpose to be sought through the use of graphical methods in elementary algebra is to lead the student to see and to appreciate the perfect harmony that exists between algebra and geometry. This harmony is revealed admirably by the fact that every algebraic equation expressing a relation between real numbers may be illustrated by means of some geometric figure called the "graph," or preferably the "locus", of the equation. And if the procedures in graphical algebra are chosen

with the view to accomplishing this central purpose to the fullest possible extent, the other worthwhile purposes will be accomplished as by-products.

If this view is correct, much time is frequently lost in graphical algebra through the failure of the teacher properly to correlate algebra and geometry. The student ought to be taught that graphical methods should be mastered not because they are to be substituted for analytical methods or to be independent of them, but because they make analytical methods more meaningful and more interesting. The implications of this point of view will be made more evident by means of the following consideration of a method of constructing the loci of first degree equations and of the use of loci in the solution of systems of such equations in x and y.

Consider the equation $2x+3y=6$. The student who has learned how to plot any point whose rectangular co-ordinates are given and how to determine approximately the rectangular co-ordinates of any given point and who has learned that the locus of an algebraic equation is the geometric figure containing those points and only those points whose co-ordinates satisfy the equation, will have little or no difficulty in seeing that the points (0,2) and (3,0) are both in the locus of this equation. And since the straight line is about the most common geometric figure, the student should naturally wish, or should be encouraged, to investigate to see whether or not the straight line determined by these two points satisfies both tests of the locus of the given equation. First, take any point on this line; call its co-ordinates x and y, respectively; and prove by the use of similar triangles that $2x+3y=6$. Then take any point not on the line, call its co-ordinates x and y, and prove that now $2x+3y$ does not equal 6. Then the locus of the equation $2x+3y=6$ must be the straight line passing through the points (0,2) and (3,0). If this equation occurs in a system with the equation $3x-4y=12$, the student can show in a similar manner that the locus of the latter equation is the straight line passing through the points (0,-3) and (4,0). Then he can determine approximately the co-ordinates of the point of intersection of the two loci and set x and y equal to these co-ordinates, respectively, and thus obtain the common solution of the two equations, or what is called the solution of the system.

In case the students studying graphical algebra have not

studied formal geometry and do not know, therefore, how to apply the fact that corresponding sides of similar triangles are proportional, it is advisable, nevertheless, for the teacher to lead them to make such application, after they have been sufficiently instructed to do so. In this way the teacher may incidentally arouse in some of the students the desire to learn more about triangles. In any case, to require or even to encourage the students to plot a large number of isolated points in the locus of a first degree equation seems to be a rather sure means of defeating the best purpose for which graphical methods should be employed in high school algebra. After the straight line locus has been constructed by the "two point" method, it is well for students, in the beginning stages of the work, to investigate to see that the line meets both tests of the locus. But after a few such exercises, the generalization should be made that the locus of every first degree equation in two variables is a straight line. Furthermore, the students should establish the habit of locating, as the two isolated points which are to be used in determining the locus, the points where the locus intersects the co-ordinate axes, since these points can usually be determined and located more quickly than any other two points of the locus. This method of constructing a straight line locus is called the intercept method. It should always be used in the general case.

In the special case of an equation of the form $ax+by=0$, the locus is determined by the origin of reference and some other point not on either of the co-ordinate axes. For examples, the locus of the equation $3x—2y=0$ passes through the origin and the point (2,3) and the locus of the equation $3x+2y$ passes through the origin and the point (2,-3). It should be noted here that in locating a point which, with the origin, will determine the locus of an equation of this form, it is very convenient to assign to x the positive value represented by the coefficient of y in the given equation, and then the corresponding value of y is obviously the coefficient of x with or without the change of sign, depending upon whether the coefficients of x and y have the same sign or opposite signs. High school students would do well to make use of this simple device when constructing the locus of an equation of the special form $ax+by=0$.

It is the firm belief of the writer that teachers who employ, in the main, the methods of teaching graphical algebra that are

herein briefly suggested are not only able to do their teaching of graphs in less time than is frequently devoted to them but are more successful in making such work interesting and meaningful and benefiicial to students. And corresponding statements could be made relative to the graphical solution of quadratic equations and of systems of quadratic equations. It is hoped, though, that these brief statements concerning the graphical solution of systems of linear equations will suffice to suggest the great possibility of correlating more effectively the analytical and the graphical methods of solution and at the same time of conserving the time and the interest of high school students and, thereby, of decreasing the mortality of students in the graphical phase of their algebraic work.

ON SIMULTANEOUS QUADRATICS

By H. L. SMITH
Louisiana State University

1. **Introduction.** It is customary to solve a pair of simultaneous, homogeneous quadratics, such as,

$$(1)\qquad \begin{cases} 2x^2-xy+y=2, \\ 3x^2+2xy-y^2=4, \end{cases}$$

by means of the substitution $y=vx$. A logically different, but algebraically equivalent, method consists in eliminating the constant term and factoring. Thus (1) may be solved as follows. First subtract the second equation from twice the first, getting

$$(2)\qquad x^2-4xy+3y^2=0.$$

Next factor (2):

$$(3)\qquad (x-y)(x-3y)=0.$$

Finally solve each of the equations

$$x-y=0,\ x-3y=0$$

simultaneouly with one of the equations (1).

It is to be noted that this method consists essentially in replacing one of the given equations by another (viz. (2)) which can be factored into linear factors. This suggests a general method for solving simultaneous quadratics which will now be explained.

2. **A theorem on factoring.** It is proved in books on analytics that the quadratic expression

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F$$

can be factored into two linear factors if, and only if,

(4) $$(B^2-AC)F+AE^2-2BED+CD^2=0.$$

3. **The general case.** Let it be required to solve the system

(5) $$\begin{cases} Ax^2+Bxy+Cy^2+2Dx+2Ey+F=0, \\ A'x^2+2B'xy+C'y^2+2D'x+2E'y+F'=0. \end{cases}$$

We assume the first equation of this system can not be factored; for if it can be, the system is easily solved. Hence equation (4) does not hold.

Add the second equation of (5) to k times the first; the result is

(6) $$(Ak+A')x^2+2(Bk+B')xy\ (Ck+C')y^2$$
$$+2(Dk+D')x+2(Ek+E')y+(Fk+F')=0.$$

Now, by § 2, the left member of (6) can be factored, if and only if,

(7) $$\{(Bk+B')^2-(Ak+A')\ (Ck+C')\}\ (Fk+F')$$
$$+(Ek+E')^2+(Dk+D')^2$$
$$-2(Bk+B')\ (Ek+E')\ (Dk+D')=0$$

Since (4) does not hold, the equation (7) is a cubic in k and hence has at least one real root. Let such a real root of (7) be substituted for k in (6). Then the left member of (6) can be factored into two linear factors and (5) may be solved by putting each of these linear factors equal to zero and solving each of the resulting equations simultaneously with either of the equations of (5).

4. **The special case of homogeneous equations.** If $D=D'=E=E'=0$, we have the special case discussed in the introduction. In this case (7) reduces to

$$\{(Bk+B')^2-(Ak+A')\ (Ck+C')\}(Fk+F')=0,$$

which has the solution $k=-F'/F$. This leads to precisely the solution there discussed.

5. **Geometric interpretation.** In plane analytics the system (5) represents a pair of conics and its solutions their points of intersection. There will, in general, be four such points of intersection. These four points determine a complete quadrangle. When a root of (7) has been substituted in (6), that equation represents a pair of opposite sides of the quadrangle. There are three such pairs of opposite sides corresponding to the three roots of (7); but any pair together with one of the conics determines the points of intersection of the two conics.

6. **An example.** Let us solve the system

$$(8)\qquad \begin{cases} 3x^2+4xy-y^2+2x-6y-2=0, \\ x^2-2xy+4y^2-4x+2y-1=0, \end{cases}$$

Here (6) becomes

(9)

$$(3k+1)x^2+2(2k-1)xy+(-k+4)y^2$$
$$+2(k-2)x+2(-3k+1)y-(2k+1)=0.$$

Hence (6) becomes after reduction

$$6k^3-3k^2+5k+4=0,$$

which has the real solution $k=-\frac{1}{2}$. Substitution of this value of k into (9) gives

$$x^2+8xy-9y^2+10x-10y=0,$$

or

$$(10)\qquad (x-y)(x+9y)+10x-10y=0.$$

The left member of (10) is only partially factored; to factor it completely, assume that (10) can be reduced to the form

$$(11)\qquad (x-y+\mathrm{L})(x+9y+\mathrm{M})=0.$$

where L, M, are to be determined. Expansion of (11) gives

$$(12)\qquad (x-y)(x+9y)+(\mathrm{L}+\mathrm{M})x+(9\mathrm{L}-\mathrm{M})y+\mathrm{LM}=0.$$

Comparison of (10) and (11) gives

$$(13)\qquad \mathrm{L}+\mathrm{M}=10,\ 9\mathrm{L}-\mathrm{M}=-10,\ \mathrm{LM}=0.$$

The equations (13) are satisfied if L=0, M=10. Hence (11) and therefore (10), may be written

$$(x-y)\ (x+9y+10)=0.$$

Finally solving each of the equations

$$x-y=0,\ x+9y+10=0$$

simultaneously with either of the equations of (8) gives the following solution of (8):

$$x=1,\ y=1;\quad x=-1/3,\ y=-1/3;$$

$$x=\frac{41+18i\sqrt{39},}{103}\qquad y=\frac{-119-2i\sqrt{39};}{103}$$

$$x=\frac{41-18i\sqrt{39},}{103}\qquad y=\frac{-119+2i\sqrt{39}.}{103}$$

Professor W. Paul Webber and Associate Professor H. L. Smith of the Louisiana State University are issuing under a joint authorship a college mathematics text entitled "Junior College Mathematics." It will be used in mimeograph form with the L. S. U. arts-and-science freshman classes during the coming session.

CONSTRUCTION OF TWO-CENTER ELLIPSE

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At times, it is desirable to construct an ellipse by means of circular arcs, having given the major and minor axes, so that it shall be a desirable approximation to the theoretical ellipse.

The present article deals with two methods of construction, the uniqueness of which lies in the fact that there are only two centers for each half, either construction giving a desirable ellipse. Fig. 1 makes use of the major and minor circles, which in Fig. 2 are only used to show how closely the approximate ellipse approaches the theoretical ellipse, four points of the latter being shown in the second quadrant.

The construction shown in Fig. 1 is due to Mr. Carl G. Barth, and is described in the Dec. 2, 1928, issue of the "American Machinist," while that shown in Fig. 2 is due to Mr. Linn and is described in "Machinery."

The basis of Fig. 1 is the mean radius of curvature. The following analysis is made for those who are interested along mathematical as well as geometrical, or graphical lines.

The radius of curvature of an ellipse at each end of the major axis is $r=\frac{b^2}{a}$ and of the minor axis, $R=\frac{a^2}{b}$, where

a=length of semi-major axis,
b=length of semi-minor axis.

The mean proportional of R and r is therefore

$R_m=\sqrt{Rr}=\sqrt{\frac{a^2}{b}\times\frac{b^2}{a}}=\sqrt{ab}$—(1), or the mean proportional of the two semi-axes

Let OA be the semi-major axis. At A erect the perpendicular AE=OA=OH, and construct a semi-circle on AE as a diameter. From B, one end of the semi-minor axis, draw a line parallel to the major axis until it intersects the semi-circle at V. Connect V with the points A and E. Then, from similar triangles,

$\frac{AB_1}{AV}=\frac{AV}{AE}$ $\therefore$ $AV=\sqrt{AB_1 x AE}=\sqrt{OB x OA}=\sqrt{ab}$, which is equal to the mean proportional of the two semi-axes, the value found in (1).

On the line AE, make AW=AV (V being found graphically as already described) and connect W and O and at the point C_1 where this line crosses the major auxiliary circle, draw the tangent line C_1D and connect the point D with the point C (C being found as shown). The line CD is a tangent to the theoretical ellipse and is a basic line of the construction. On CD make ZN=ZA and draw NQ perpendicular to CD; this perpendicular locates one of the desired radii.

To locate the other radius, make BS=AQ, and connect S with Q, and make the angle QST=angle SQT, thus locating the point T. Extend the line TQ, making QX=QA. The point X will be the intersection of the arcs XA and XB; the arc XA being struck with QA as a radius and the arc XB being struck with TB as a radius.

Mr. Barth says that this method gives better results than any other method to his knowledge when applied to fairly large eccentricities. The writer tried it with a ratio of the major axis to the minor axis =4:1, and found it excellent. The ratio shown in Fig. 1 is 4:2. He has also called my attention to the fact that CD is equal to the semi-minor axis and the extension of CD to the minor axis is equal to the semi-major axis and that this holds true for any ratio of the two axes.

In Fig. 2, lay off the major and minor axes, and connect A and D. Make DF=DE and bisect AF, and at H the midpoint erect the perpendicular which will cut AO at K and EO extended at L. KM=KA. M is the junction point of the arcs AM and DM. KA is the radius of the arc AM and LD is the radius of the arc DM. This method described by Mr. Linn gives excellent results. The illustration has a ratio of the axes=3.875:2.5—this oddness being due to the desire to make the illustration as large as possible.

Professor H. E. Buchanan, of Tulane University, has recently been honored. The degree of Doctor of Laws was conferred on him by the University of Arkansas, which we understand is his alma mater.